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Idealized Operation of the Class E Tuned Power Amplifier

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***Abstract*—The class E tuned power amplifier consists of a load network and a single transistor that is operated as a switch at the carrier frequency of the output signal. The most simple type of load network consists of a capacitor shunting the transistor and a series-tuned output circuit, which may have a residual reactance. Circuit operation is determined by the transistor when it is on, and by the transient response of the load network when the transistor is off. The basic equations governing amplifier operation are derived using Fourier series techniques and a high-Q assumption. These equations are then used to determine component values for optimum operation at an efficiency of 100 percent. Other combinations of component values and duty cycles which result in 100-percent efficiency are also determined. The harmonic structure of the collector voltage waveform is analyzed and related amplifier configurations are discussed. While this analysis is directed toward the design of high-efficiency power amplifiers, it also provides insight into the operation of modern solid-state VHF-UHF tuned power amplifiers.**

I. Introduction

The "class E" concept recently introduced by the Sokals [1], [2] offers a new means of highly efficient power amplification. This paper expands upon the Sokals' work by providing an analytical basis for class E operation and by deriving additional amplifier configurations. Before entering the technical discussion, some definitions must be clarified.

As used here, "class E" refers to a tuned power amplifier composed of a single-pole switch and a load network. The switch consists of a transistor[1] or combination of transistors and diodes that are driven on and off at the carrier frequency of the signal to be amplified. In its most basic form, analyzed here, the load network consists of a resonant circuit in series with the load, and a capacitor which shunts the switch (Fig. 1(a)). Note that the total shunt capacitance is due to what is inherent in the transistor (C1) and added by the load network (C2). The collector voltage waveform is then determined by the switch when it is on, and by the transient response of the load network when the switch is off.

Class E amplification is easily differentiated from other classes of power amplification. Classes A, B, and C refer to amplifiers in which the transistors act as current sources; sinusoidal collector voltages are maintained by the parallel-tuned output circuit. If the transistors are driven hard enough to saturate, they cease to be current sources; however, the sinusoidal collector voltage remains. Classes D and S are characterized by two (or more) pole switching configurations that define either a voltage or current waveform without regard for the load network. Class D employs bandpass filtering, while class S employs low-pass filtering[2] (with respect to the carrier or switching frequency). This author has used class F to designate multiple-resonator power amplifiers in which the transistor acts as a (possibly saturating) current source, and a collector voltage waveform consisting of a sum of sinusoids is maintained by the output circuit. Further discussion and comparisons are given elsewhere [1], [3] and in Table I.

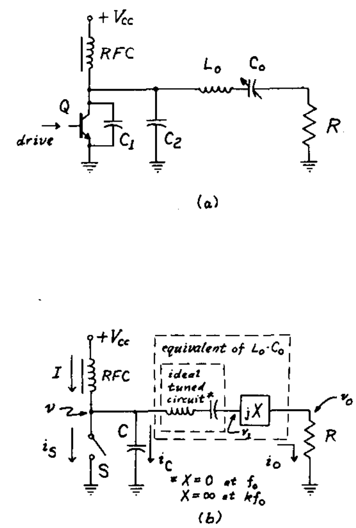

Fig. 1. Class E amplifier. (a) Basic circuit. (b) Equivalent circuit.

The definition of class E operation used by the Sokals [1], [2] allowed a variety of circuit topologies containing a switch and a load network, and stated three specific objectives for the collector voltage and current waveforms: a) the rise of the voltage across the transistor at turn-off should be delayed until after the transistor is off, b) the collector voltage should be brought back to zero at

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[1]Since bipolar transistors represented the state of the art when this paper was written, "transistor" is often used here in place of "active device." The theory is generally applicable to vacuum tubes, field-effect transistors, and other devices by substitution of analogous terms (e.g., "drain current" for "collector current.")

[2]Some authors use "class D" and "class S" interchangeably, while others reverse the author's definitions.

TABLE I
COMPARISON OF DIFFERENT AMPLIFIERS PRODUCING SINEWAVE OUTPUT

Class	P_{max}	%	Comments
A	0.125	50	360° conduction angle
B	0.125	78.5	180° conduction angle
C	0.0981	89.6	120.6° conduction angle
	0.0000	100	0° conduction angle
D	0.318	100	uses two devices with 1 A peak current
E	0.0981	100	optimum-50% duty cycle
F	0.159	100	infinite number of resonators
HiLo	0.203	81.8	Efficiency is fundamental frequency output/dc input [12]
S	0.125	100	Ac coupled output

Note: All cases assume ideal transistors: zero saturation voltage, zero saturation resistance, infinite off resistance, and zero switching time.

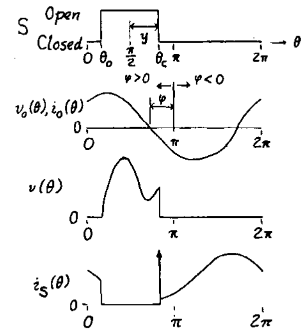

Fig. 2. Waveforms in class E amplifier.

the time of transistor turn-on, and c) the slope of the collector voltage should be zero at the time of turn-on. This paper broadens the definition of class E as follows. 1) An amplifier containing a switch and a load network and meeting criteria a), b), and c) is called "optimum class E." An amplifier containing a switch and a load network, but *not* meeting these three criteria is called "suboptimum class E." This definition allows an amplifier which is mistuned or not optimized [4] to be described as class E. (For operation with real transistors, their nonzero saturation voltage replaces the zero voltage in objective b).).

All class E power amplifiers (as well as class D and saturating class C power amplifiers) might more appropriately be called power converters. In these circuits, the driving signal causes switching of the transistor, but there is no relationship between the amplitudes of the driving signal and the output signal. Nonetheless, these circuits are colloquially referred to as "power amplifiers," and it is possible to define a power gain and to note that the frequency and phase of the output signal track those of the driving signal. The colloquial term "power amplifier" is more readily recognized in radio-frequency applications, and is therefore used in this paper.

II. BASIC EQUATIONS

The characteristics of a class E power amplifier can be determined by finding its steady-state waveforms. The least complex configuration includes a single transistor switch, a shunt capacitor, a series-tuned output circuit, and a RF choke (Fig. 2). A driving waveform capable of producing the desired switching action is assumed. This driving waveform includes the frequency and phase modulation desired in the output signal, but not the amplitude modulation. (Amplitude modulation may be accomplished by variation of the collector supply voltage.)

A simple equivalent circuit of this amplifier is based on the following five assumptions.

The RF choke allows only a constant (dc) input current and has no series resistance.

The Q of the series-tuned output circuit is high enough that the output current is essentially a sinusoid at the carrier frequency.

The switching action of the transistor is instantaneous and lossless (except when discharging the shunt capacitor); the transistor has zero saturation voltage, zero saturation resistance, and infinite off resistance.

The total shunt capacitance is independent of the collector voltage.

The transistor can pass negative current and withstand negative voltage. (This is inherent in MOS devices, but requires a combination of bipolar transistors and diodes.)

This equivalent circuit is shown in Fig. 1(b). The series reactance jX is actually produced by the difference in the reactances of the inductor and capacitor of the series-tuned circuit. Note that the jX reactance applies *only* to the fundamental frequency; the reactance is assumed to be infinite at harmonic frequencies. The voltage $v_1(\theta)$ is actually fictitious; however, it is a convenient reference point to use in the analysis.

Analysis of the class E amplifier is straightforward but quite tedious. There is no clear source of voltage or current, as in classes A, B, C, and D amplifiers. The collector voltage waveform is a function of the current charging the capacitor, and the current is a function of the voltage on the load, which is in turn a function of the collector voltage. All parameters are interrelated. The analysis begins by determining the collector voltage waveform as a function of the dc input current and the sinusoidal output current. Next, the fundamental frequency component of the collector voltage is related to the output current, and the dc component of the collector voltage is related to the supply voltage. These relationships result in a nonlinear equation which can be solved analytically or numerically. Finally, input and output power and efficiency can be calculated.

A. Basic Relationships

The output voltage and current are sinusoidal and have the forms

$$v_0(\theta)=c\sin(\omega t+\varphi)=c\sin(\theta+\varphi) \tag{2.1}$$

and

$$i_0(\theta)=\frac{c}{R}\sin(\theta+\varphi) \tag{2.2}$$

where θ is an "angular time" used for mathematical convenience. The parameters c and φ are to be determined; φ is defined in Fig. 2.

The hypothetical voltage $v_1(\theta)$ is also a sinusoid, but has a different phase because of reactance X

$$v_1(\theta)=v_0(\theta)+v_X(\theta) \tag{2.3}$$

$$=c\sin(\theta+\varphi)+X\frac{c}{R}\cos(\theta+\varphi) \tag{2.4}$$

$$=c_1\sin(\theta+\varphi_1) \tag{2.5}$$

where

$$c_1=c\sqrt{1+\frac{X^2}{R^2}}=\rho c \tag{2.6}$$

and

$$\varphi_1=\varphi+\psi=\varphi+\tan\left(\frac{X}{R}\right). \tag{2.7}$$

Since the RF choke forces a dc input current and the series-tuned output circuit forces a sinusoidal output current, the difference between those two currents must flow into (or out of) the switch–capacitor combination. When switch S is open, the difference flows into capacitor C; when switch S is closed, the current difference flows through switch S. If a capacitor voltage of other than zero volts is present at the time the switch closes, the switch discharges that voltage to zero volts, dissipating the stored energy of $\frac{1}{2}CV^2$.

For purposes of this analysis, the discharge of the capacitor may be assumed to be a current impulse $q\delta(\theta-\theta_c)$, where θ_c is the time at which the switch closes. However, in a real amplifier, discharge of the capacitor requires a nonzero length of time, during which the collector voltage and current are simultaneously nonzero. In any case, however, the total dissipation is the energy stored in the capacitor, and does not depend on the particulars of the discharge waveforms. The model remains valid as long as the time required to discharge the capacitor is a relatively small fraction of the RF cycle.

When S is off, the collector voltage is produced by the charging of capacitor C by the difference current, hence,

$$v(\theta)=\frac{1}{\omega C}\int_{\theta_0}^{\theta}i_c(u)\,du \tag{2.8}$$

where θ_0 indicates the time at which S opens.

Since the nonzero collector voltage appears when the transistor is off, it is convenient to describe the waveforms in terms of the half off-time y, which is in radians. The center of the off-time is arbitrarily defined as $\pi/2$ (Fig. 2). The switching instants are now $\theta_0=\pi/2-y$ and $\theta_c=\pi/2+y$. Note that in the event that $\theta_0<0$ and is therefore outside of the 2π interval corresponding to one cycle, it can be repaced by $\theta_0+2\pi$, with appropriate modification to the integral in (2.8). Equivalently, the interval of a cycle can be redefined when necessary. The collector voltage at time θ can now be evaluated by expanding (2.8)

$$v(\theta)=\frac{1}{B}\int_{(\pi/2)-y}^{\theta}\left[I-\frac{c}{R}\sin(u+\varphi)\right]du \tag{2.9}$$

$$=\left[\frac{I}{B}\left(-\frac{\pi}{2}+y\right)+\frac{c}{BR}\sin(\varphi-y)\right]$$
$$+\frac{I}{B}\theta+\frac{c}{BR}\cos(\theta+\varphi) \tag{2.10}$$

where

$$B=\omega C. \tag{2.11}$$

Since the tuned circuit has zero impedance to fundamental frequency current, there can be no fundamental frequency voltage drop across it. This means that the fundamental frequency component of the collector voltage waveform must be the hypothetical voltage $v_1(\theta)$. The magnitude of this component can be calculated by a Fourier integral. Unfortunately, the collector voltage is not symmetrical around $\pi/2$, which makes the phase φ an unknown.

B. Fourier Analysis

The magnitude c_1 of the fundamental frequency component of the collector voltage is then

$$c_1=\frac{1}{\pi}\int_0^{2\pi}v(\theta)\sin(\theta+\varphi_1)\,d\theta \tag{2.12}$$

$$=-2\left[\frac{I}{\pi B}\left(\frac{\pi}{2}-y+\varphi_1\right)+\frac{c}{\pi BR}\sin(y-\varphi)\right]$$
$$\cdot\cos\varphi_1\sin y+\frac{I}{B}\Big[-2\sin\varphi_1\sin y$$
$$+2\left(\frac{\pi}{2}+\varphi_1\right)\cos\varphi_1\sin y+2y\sin\varphi_1\cos y\Big]$$
$$-\frac{c}{2\pi BR}\left[\sin(2\varphi+\psi)\sin 2y-2y\sin\psi\right]. \tag{2.13}$$

It is now possible to solve for c by substituting ρc for c_1 and collecting terms

$$\rho c+c\left[\frac{\sin(2\varphi+\psi)\sin 2y-2y\sin\psi}{2\pi BR}+\frac{2\sin(y-\varphi)\cos\varphi_1\sin y}{\pi BR}\right]$$
$$=\frac{I}{\pi B}\left[(-\pi+2y-2\varphi_1+\pi+2\varphi_1)\cos\varphi_1\sin y\right.$$
$$\left.+(2y\cos y-2\sin y)\sin\varphi_1\right]. \tag{2.14}$$

Thus

$$c=IR\frac{2y\sin y\cos\varphi_1+(2y\cos y-2\sin y)\sin\varphi_1}{\pi BR\rho+\frac{1}{2}\sin(2\varphi+\psi)\sin 2y-y\sin\psi+2\sin(y-\varphi)\cos\varphi_1\sin y} \tag{2.15}$$

$$=IRh(\varphi,\psi,y,B,R,\rho). \tag{2.16}$$

This relationship will be useful later in finding φ. Since the fundamental frequency component of the collector voltage is by definition a sinewave of phase φ_1, there can be no cosine or quadrature component with respect to phase φ_1. A second relationship among the parameters is then found from

$$0=\frac{1}{\pi}\int_0^{2\pi}v(\theta)\cos(\theta+\varphi_1)\,d\theta \tag{2.17}$$

$$\begin{aligned}&=\left[\frac{I}{\pi B}\left(y-\frac{\pi}{2}-\varphi_1\right)+\frac{c}{\pi BR}\sin(\varphi-y)\right]\\&\cdot[-2\sin\varphi_1\sin y]+\frac{I}{\pi B}\Big[-2\cos\varphi_1\sin y\\&-2\left(\frac{\pi}{2}+\varphi_1\right)\sin\varphi_1\sin y+2y\cos\varphi_1\cos y\Big]\\&-\frac{c}{2\pi BR}\sin 2y\cos(2\varphi+\psi)+\frac{yc\cos\psi}{\pi BR}.\end{aligned} \tag{2.18}$$

Now c can be separated as before, producing

$$c=IR\frac{[2y\sin\varphi_1\sin y-2y\cos\varphi_1\cos y+2\cos\varphi_1\sin y]}{\left[-2\sin(\varphi-y)\sin y\sin\varphi_1-\frac{1}{2}\sin 2y\cos(2\varphi+\psi)+y\cos\psi\right]} \tag{2.19}$$

$$=IRg(\varphi,\psi,y). \tag{2.20}$$

The similiarity of (2.16) and (2.20) requires that

$$g(\varphi,\psi,y)=h(\varphi,\psi,y,B,R,\rho). \tag{2.21}$$

When component values and switch duty cycle have been specified, φ is the only unknown in (2.21). Finding the operating parameters of a class E amplifier then begins by finding φ. It is possible to solve (2.21) analytically in most cases; solutions for arbitrary component values and duty cycle are given in [5]. In the event that an analytical solution is difficult or impossible, one can resort to a brute force numerical solution to find φ, as in the author's first approach [6].

C. Power and Efficiency

The relationship of the supply voltage V_{cc} to the other parameters has yet to be determined. It can be found by observing that there is no dc voltage drop across the RF choke, thus using (2.10) and (2.20), and abbreviating $g(\varphi,\psi,y)$ as g

$$V_{cc}=\frac{1}{2\pi}\int_0^{2\pi}v(\theta)\,d\theta \tag{2.22}$$

$$V_{cc}=\frac{I}{2\pi B}\int_{(\pi/2)-y}^{(\pi/2)+y}\left\{\left[y-\frac{\pi}{2}+g\sin(\varphi-y)\right]+\theta+g\cos(\theta+\varphi)\right\}d\theta \tag{2.23}$$

$$=\frac{I}{2\pi B}\left\{[2y^2+2yg\sin(\varphi-y)]-2g\sin\varphi\sin y\right\} \tag{2.24}$$

$$=IR_{\mathrm{dc}}. \tag{2.25}$$

R_{dc} is the resistance (load) that the amplifier shows to the power supply. It is now possible to compute input and output power and efficiency. The output power is

$$P_0=\frac{1}{2}\frac{c^2}{R}=\frac{I^2g^2R}{2}=\frac{V_{cc}^2g^2R}{2R_{dc}^2}. \tag{2.26}$$

The input power is simply

$$P_i=V_{cc}I=\frac{V_{cc}^2}{R_{dc}} \tag{2.27}$$

so the efficiency is

$$\eta=\frac{P_0}{P_i}=\frac{g^2}{2}\frac{R}{R_{dc}}. \tag{2.28}$$

D. Device Stress

To determine the peak collector voltage, it is first necessary to determine the time at which it occurs. This is accomplished by differentiating the waveform and setting the result equal to zero. Since $I=V_{cc}/R_{dc}$, and

$$c=IRg=\frac{V_{cc}}{R_{dc}}Rg \tag{2.29}$$

the collector voltage waveform may be described by

$$v(\theta)=\frac{V_{cc}}{R_{dc}B}\left[\left(y-\frac{\pi}{2}\right)+\theta+g\sin(\varphi-y)+g\cos(\theta+\varphi)\right]. \tag{2.30}$$

Now

$$0=\left.\frac{dv(\theta)}{d\theta}\right|_{\theta=\theta_{v\max}}=\frac{V_{cc}}{R_{dc}B}\cdot[1-g\sin(\theta_{v\max}+\varphi)]. \tag{2.31}$$

From this

$$\theta_{v\max}=\arcsin\frac{1}{g}-\varphi \tag{2.32}$$

and the peak voltage v max can then be found by using $\theta_{v\,\max}$ in (2.30). In the event that there is a negative collector voltage, its peak value can be determined similarly using

$$\theta_{v\,\min}=\pi-\arcsin\frac{1}{g}-\varphi \tag{2.33}$$

provided that it occurs before the time of turn-on.

The peak collector current is more easily determined, since it must occur at a peak of the output current. Thus

$$i_{S\,\max}=\frac{c}{R}+I=\frac{IRg}{R}+I=I(1+g). \tag{2.34}$$

In the event that the peak output current occurs outside of the interval in which the transistor is turned on, (2.34) no longer applies, and $i_{S\,\max}$ is the value at either turn-on or turn-off. One should be careful, however, to note that unless the voltage on the capacitor is zero at time of turn-in, $i_{S\,\max}$ is in theory infinite, and in practice considerably larger than that given by (2.34).

The power output capability $P_{\max}$ provides a means of comparing different designs (values of B and ψ) with each other and with other types of amplifiers. It is convenient to define this [3] as the output power produced when the device has a peak collector voltage of 1 V and a peak collector current of 1 A. It may be determined from any set of values based on the same supply voltage and load according to

$$P_{\max}=\frac{P_0}{v_{\max}\, i_{S\,\max}}. \tag{2.35}$$

If the capacitor voltage is not zero at the time the switch closes, $P_{\max}$ will be zero in theory and greatly reduced from that given by (2.35) in practice.

III. High-Efficiency Operation

The ideal-circuit model of the class E amplifier used in this paper contains only lossless elements (other than the load). Consequently, the only loss mechanism is discharge of the shunt capacitor as the transistor is turned on. High-(unity) efficiency operation will then be possible if the circuit parameters can be chosen to cause the collector voltage to drop to zero at the instant the transistor turns on. The slope of the collector voltage at turn-on and the pulse width (duty cycle) of the transistor switch are two design options, in addition to the shunt capacitance and the reactance of the series-tuned circuit. Realizability of such designs will be shown by the derivation of real elements and waveforms. Component values for optimum operation will be derived and trade-offs will then be discussed.

A. Derivation of Equations

The requirement for an efficiency of 100 percent can now be expressed by setting (2.30) equal to zero at time $\theta=\pi/2+y$, which produces

$$0=2y-2g\cos\varphi\sin y. \tag{3.1}$$

Thus

$$\cos\varphi=\frac{y}{g\sin y} \tag{3.2}$$

which is the first constraint on g and φ.

The slope of the waveform at the time of turn-on must also be defined. Letting ζ represent the normalized slope results in

$$\zeta=\frac{1}{V_{cc}}\frac{dv(\theta)}{d\theta}\bigg|_{\theta=\frac{\pi}{2}+y} \tag{3.3}$$

$$=\frac{1}{R_{dc}B}\left[1-g\cos(y+\varphi)\right]. \tag{3.4}$$

Rearrangement of (2.24) produces

$$R_{dc}B=\frac{2y^2+2yg\sin(\varphi-y)-2g\sin\varphi\sin y}{2\pi}. \tag{3.5}$$

Substitution of this into (3.4) followed by expansion of the trigonometric functions, and rearrangement to separate $\cos\varphi$ and $\sin\varphi$ produces

$$\begin{aligned}&[\pi\cos y-\zeta y\sin y]\cos\varphi\\ &\qquad+[-\pi\sin y+\zeta y\cos y-\zeta\sin y]\sin\varphi\\ &=\frac{\pi-\zeta y^2}{g}.\end{aligned} \tag{3.6}$$

Selection of a value of ζ then forms a second constraint on the values of g and φ.

Since there are now two constraints on the two unknowns, g and φ, a solution should be possible. This is accomplished by dividing the left and right sides of (3.6) by the left and right sides of (3.5), respectively,

$$\begin{aligned}&[\pi\cos y-\zeta y\sin y]\\ &\qquad+[-\pi\sin y+\zeta y\cos y-\zeta\sin y]\tan\varphi\\ &=(\pi-\zeta y^2)\frac{\sin y}{y}.\end{aligned} \tag{3.7}$$

Rearranging this,

$$\tan\varphi=\frac{\dfrac{\sin y}{y}-\cos y}{\dfrac{\zeta y}{\pi}\cos y-\left(1+\dfrac{\zeta}{\pi}\right)\sin y}. \tag{3.8}$$

Once φ has been determined, g can be found from (3.2)

$$g=\frac{y}{\cos\varphi\sin y}. \tag{3.9}$$

Substitution of the efficiency of unity in (2.29) and rearrangement yields

$$R_{dc}=\frac{g^2R}{2}. \tag{3.10}$$

Substitution of this into (3.5) then produces

$$B=\frac{2y^2+2yg\sin(\varphi-y)-2g\sin\varphi\sin y}{\pi g^2}R \tag{3.11}$$

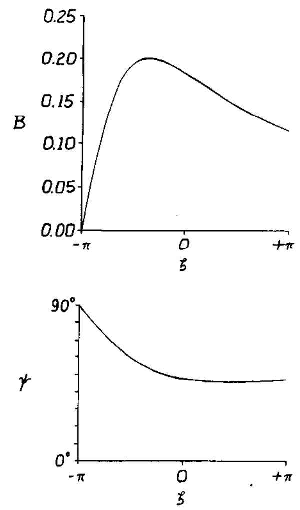

Fig. 3. Circuit element values for 100-percent efficiency with 50-percent duty cycle ($R=1$).

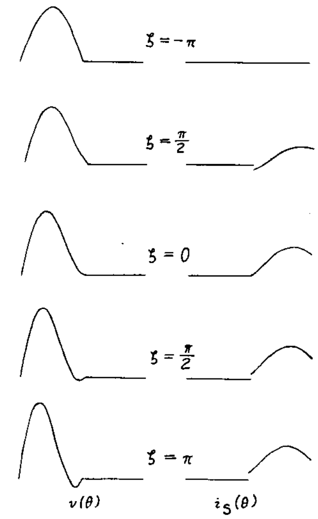

Fig. 4. Waveforms as function of slope with 50-percent duty cycle.

which is used to obtain the value of the capacitor shunting the switch. In an actual application, the capacitance inherent in the transistor would comprise some of the shunt capacitance.

By recalling (2.21),

$$g=\frac{2y\sin\varphi_1\sin y-2y\cos\varphi_1\cos y+2\cos\varphi_1\sin y}{-2\sin(\varphi-y)\sin y\sin\varphi_1-\frac{1}{2}\sin 2y\cos(2\varphi+\psi)+y\cos\psi}. \tag{3.12}$$

Since the value of g has been determined, the only unknown is the load phase angle, which appears as $\varphi_1=\varphi+\psi$. To determine ψ, it is necessary to expand the terms containing φ, so that $\sin\psi$ and $\cos\psi$ can be separated. For convenience, let

$$q_1=-2g\sin(\varphi-y)\sin y-2y\sin y \tag{3.13}$$

$$q_2=2y\cos y-2\sin y \tag{3.14}$$

and

$$q_3=-\frac{g}{2}\sin 2y. \tag{3.15}$$

Then (3.12) becomes

$$\begin{aligned}q_1(\sin\varphi\cos\psi+\cos\varphi\sin\psi)&\\ +q_2(\cos\varphi\cos\psi-\sin\varphi\sin\psi)&\\ +q_3(\cos 2\varphi\cos\psi-\sin 2\varphi\sin\psi)&\\ +gy\cos\psi=0.&\end{aligned} \tag{3.16}$$

To solve for ψ, it is necessary to separate $\sin\psi$ and $\cos\psi$ terms. Thus

$$(\sin\psi)[q_1\cos\varphi-q_2\sin\varphi-q_3\sin 2\varphi]$$
$$+\cos\psi[q_1\sin\varphi+q_2\cos\varphi+q_3\cos 2\varphi+gy]=0 \tag{3.17}$$

which can be rearranged to produce

$$\tan\psi=\frac{\sin\psi}{\cos\psi}=\frac{q_n}{q_d}=\frac{q_1\sin\varphi+q_2\cos\varphi+q_3\cos 2\varphi+gy}{q_2\sin\varphi+q_3\sin 2\varphi-q_1\cos\varphi}. \tag{3.18}$$

Finally,

$$\psi=\arctan(q_n/q_d). \tag{3.19}$$

Note that this is a two quadrant inverse tangent, since ψ must be between $-\pi/2$ and $+\pi/2$.

B. Results

Since high-efficiency operation is possible with a variety of values of the turn-off slope ζ and duty cycle y, the designer has some options at his or her disposal. The effects of the choice of ζ and y are illustrated by two examples. Component values for optimum operation will then be derived.

A 50-percent duty cycle ($y=\pi/2$) is used in the first example, along with $R=1$ and $V_{cc}=1$. Values of B and ψ for an efficiency of 100 percent with a specified value of ζ are shown in Fig. 3. The resultant collector voltage and current waveforms are then shown in Fig. 4. For a 50-percent duty cycle, values of ζ less than $-\pi$ are not realizable because they require a negative capacitance in (3.11). Large positive values of ζ are realizable, and produce large positive and negative excursions of the collector voltage waveform. Fig. 5 depicts the power output P_0 and power output capability $P_{\max}$ as functions of ζ. Both decrease toward zero rapidly as ζ approaches $-\pi$. As ζ becomes a large positive value, P_0 increases slowly, while $P_{\max}$ decreases slowly.

The second example illustrates the effects of choosing different duty cycles by fixing the slope ζ at 0. Values of B and ψ for a 100-percent efficiency are shown in Fig. 6, while the corresponding waveforms are shown in Fig. 7.

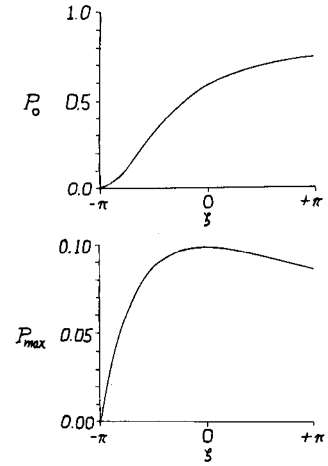

Fig. 5. Power output as function of slope ($R=1$, $V_{cc}=1$).

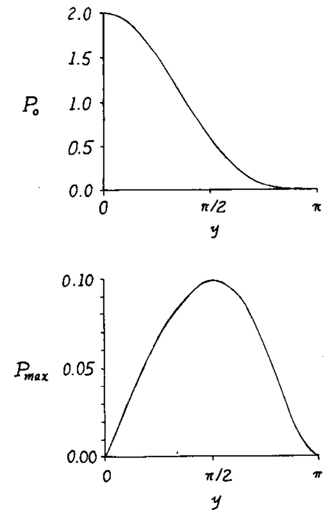

Fig. 8. Power output as function of duty cycle ($R=1$, $V_{cc}=1$).

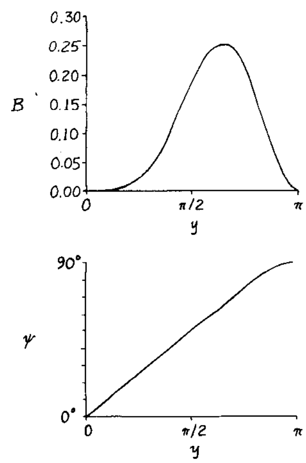

Fig. 6. Circuit element values for 100-percent efficiency with different duty cycles ($R=1$, $\zeta=0$).

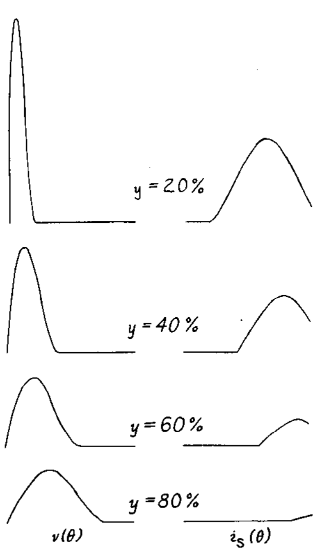

Fig. 7. Waveforms as function of duty cycle ($\zeta=0$).

These values and waveforms show realizability, although values for nearly 100 or 0 percent duty cycles may not be practical. Power output and capability are shown in Fig. 8. While the output with a given load line is maximum at an (almost) zero duty (off) cycle, the maximum capability occurs at a 50-percent duty cycle.

It is apparent from the above examples that the optimum operating point has a 50-percent duty cycle and a slope of 0. This combination not only produces the peak power output capability for a given device, but also eliminates both negative collector voltage and negative collector current. Component values and device stresses for this optimum mode of operation can be derived from the previous equations. From (3.8),

$$\tan\varphi = -\frac{2}{\pi} \tag{3.20}$$

from which

$$\varphi = -32.482° = -0.56691 \text{ rad.} \tag{3.21}$$

Consideration of trigonometric relationships shows that

$$\sin\varphi = \frac{-2}{\sqrt{4+\pi^2}} = \frac{-1}{\sqrt{1+\pi^2/4}} \tag{3.22}$$

hence from (3.9),

$$g = \sqrt{1+\pi^2/4} = 1.8621. \tag{3.23}$$

From (3.10) and (3.11),

$$R_{dc} = \frac{(1+\pi^2/4)}{2} R = 1.7337R \tag{3.24}$$

and

$$B = \frac{2}{(1+\pi^2/4)R} = \frac{1}{5.4466R}. \tag{3.25}$$

Using (3.22) and similarly obtaining

$$\cos\varphi = \frac{\pi}{\sqrt{\pi^2+4}} \tag{3.26}$$

(3.13)–(3.15) and (3.18) produce

$$\psi = \arctan\left[\frac{\pi}{8}\left(\frac{\pi^2}{2}-2\right)\right] = 49.052^\circ = 0.85613 \text{ rad.} \quad (3.27)$$

The series reactance is then

$$X = R\tan\psi = 1.1525R. \quad (3.28)$$

The output voltage, obtained from (2.31), is

$$c = \frac{2}{\sqrt{1+\pi^2/4}} V_{cc} \approx 1.074 V_{cc} \quad (3.29)$$

and the output power may be obtained from the above or (3.24). Substitution of these values into (2.32), (2.30), and (2.34) produces a peak voltage of $3.56V_{cc}$ and a peak current of $2.84I$. This results in $P_{\max} = 0.0981$ (for class B and class D, $P_{\max}$ is 0.125 and 0.318, respectively). This "optimum class E" amplifier is the case described by the Sokals [1], [2], and values derived here agree with their values, within measurement accuracy.

IV. Harmonic Structure

Knowledge of the harmonic structure of the collector voltage waveform is necessary in the design of a practical class E amplifiers. As in other amplifiers, the tuned output circuit must have a sufficiently high Q that harmonic currents are kept to insignificant levels as far as amplifier operation and efficiency are concerned. However, reduction of harmonics to levels acceptable in a radio transmitter (e.g., -60 dB relative to the carrier) generally requires additional filtering. This filter is placed between the series-tuned output circuit and the load [7].

A harmonic voltage component of $v(\theta)$ may be represented as $c_n \sin(n\theta+\varphi_n) = a_n \cos n\theta + b_n \sin n\theta$. These components may be evaluated by Fourier integrals similar to those in Section II, producing $a_0 = V_{cc}$,

$$a_n = \frac{V_{cc}}{R_{dc}B}\left\{\left[y-\frac{\pi}{2}+g\sin(\varphi-y)\right]q_2+q_4+gq_6\right\} \quad (4.1)$$

and

$$b_n = \frac{V_{cc}}{R_{dc}B}\left\{\left[y-\frac{\pi}{2}+g\sin(\varphi-y)\right]q_1+q_3+gq_5\right\}. \quad (4.2)$$

The quantities q_i correspond to the sine or cosine integrals of the dc, ramp, and cosine components of $v(\theta)$

$$q_1 = \begin{cases} \frac{2}{n\pi}(-1)^{(n-1)/2}\sin ny, & n \text{ odd} \\ 0, & n \text{ even} \end{cases} \quad (4.3)$$

$$q_2 = \begin{cases} 0, & n \text{ odd} \\ \frac{2}{n\pi}(-1)^{n/2}\sin ny, & n \text{ even} \end{cases} \quad (4.4)$$

$$q_f3 = \begin{cases} (-1)^{(n-1)/2}\frac{\sin ny}{n}, & n \text{ odd} \\ \frac{2}{n\pi}(-1)^{n/2}\left[\frac{\sin ny}{n}-y\cos ny\right], & n \text{ even} \end{cases} \quad (4.5)$$

$$q_4 = \begin{cases} \frac{2}{n\pi}(-1)^{(n+1)/2}\left(\frac{\sin ny}{n}+y\cos ny\right), & n \text{ odd} \\ (-1)^{n/2}\frac{\sin ny}{n}, & n \text{ even} \end{cases} \quad (4.6)$$

$$q_5 = \begin{cases} -\frac{\sin\varphi}{\pi}\left[\frac{\sin 2y}{2}+y\right], & n=1 \\ \frac{(-1)^{(n+1)/2}\sin\varphi}{\pi}\left[\frac{\sin[(n+1)y]}{n+1}+\frac{\sin[(n-1)y]}{n-1}\right], & n \text{ odd}, >1 \\ \frac{(-1)^{n/2}\cos\varphi}{\pi}\left[\frac{\sin[(n+1)y]}{n+1}-\frac{\sin[(n-1)y]}{n-1}\right], & n \text{ even} \end{cases} \quad (4.7)$$

$$q_6 = \begin{cases} -\frac{\cos\varphi}{\pi}\left[\frac{\sin 2y}{2}-y\right], & n=1 \\ \frac{(-1)^{(n+1)/2}\cos\varphi}{\pi}\left[\frac{\sin[(n+1)y]}{n+1}-\frac{\sin[(n-1)y]}{n-1}\right], & n \text{ odd}, >1 \\ \frac{(-1)^{-(n+2)/2}\sin\varphi}{\pi}\left[\frac{\sin[(n+1)y]}{n+1}+\frac{\sin[(n-1)y]}{n-1}\right], & n \text{ even.} \end{cases} \quad (4.8)$$

The harmonic voltage spectrum of an optimum class E amplifier is shown in Fig. 9. The amplitudes of the harmonic voltages decreases as $1/n^2$, as in a class B amplifier. This is to be expected, since the optimum amplifier has a continuous collector voltage waveform. Mistuning the amplifier produces a collector voltage

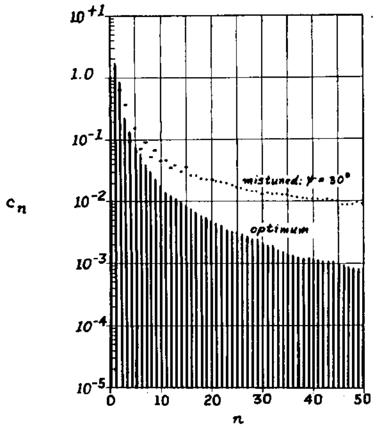

Fig. 9. Harmonic structure.

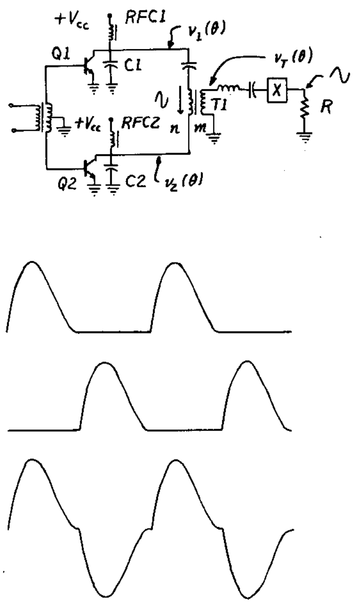

Fig. 10. Push-pull class E amplifier.

waveform with a step discontinuity (the abrupt capacitor discharge). The harmonic spectrum of a mistuned ($\psi = 30°$) optimum amplifier [5] is also shown in Fig. 9. As expected, these harmonics decrease as $1/n$.

V. Other Configurations

Other class E configurations employing a single transistor switch and a single reactive storage element are possible, and can be analyzed by the same equations, given proper analogies. Three such configurations are discussed here.

A. Push-Pull Class E

The push-pull configuration offers a means of combining two class E amplifiers to obtain a larger power output. Fig. 10 depicts this configuration operating optimally. A

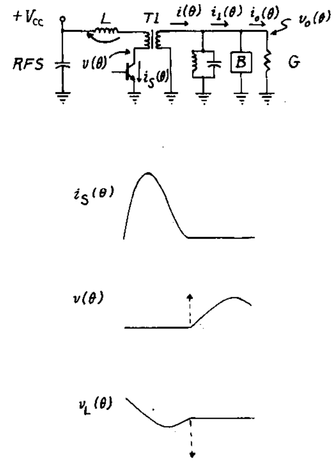

Fig. 11. Parallel-tuned class E amplifier with series inductor.

similar configuration operating suboptimally is shown in [4]. As in any push-pull amplifier, the two devices are driven with opposite phases. Each, however, operates as if it were a single transistor class E amplifier. When a given transistor is driven on, it provides a ground connection on the primary of the output transformer, causing the dc input current and transformed sinusoidal output current to charge the capacitor shunting the other transistor.

The voltage appearing on the secondary winding of the output transformer contains both a positive and negative "class E" shape. Consequently, it has a carrier frequency component which has twice the amplitude of the carrier frequency component of either collector waveform. The resulting impedance seen by either half of the amplifier looking into the primary winding of the transformer is then

$$Z_{se} = \frac{1}{2}\frac{m^2}{n^2}Z_0. \tag{5.1}$$

Otherwise, the equations describing single-ended operation apply to push-pull operation as well.

B. Parallel-Tuned Amplifier with Series Inductor

A parallel-tuned class E amplifier employing a series inductor as the storage element is shown in Fig. 11. The series inductor might be discrete, or could be the effect of the transformer. This configuration is the *dual* of the series-tuned amplifier with a shunt capacitor discussed previously in this paper. Here the roles of voltage and current are reversed. The analogies are straightforward and therefore given in Table II without derivation. As a consequence of the duality, the net effect of the parallel-tuned circuit on the fundamental frequency must be a capacitive susceptance for optimum operation.

C. Parallel-Tuned Amplifier with Series Capacitor

A third configuration is the parallel-tuned class E amplifier with a series capacitor (Fig. 12). While this

TABLE II
ANALOGOUS PARAMETERS FOR PARALLEL-TUNED SERIES-INDUCTOR AMPLIFIER

Series-tuned/shunt C	Parallel-tuned/series L
$B = \omega C$	$X = \omega L$
$X = \omega L$	$B = \omega C$
R	$G = 1/R$
φ	φ
$\psi = \tan\left(\frac{X}{R}\right)$	$\psi = \tan\left(\frac{B}{G}\right)$
$v(\theta)$	$i_S(\theta)$
$i_S(\theta)$	$v(\theta)$
c	c
V_{cc}	I
I	V_{cc}
R_{dc}	G_{dc}

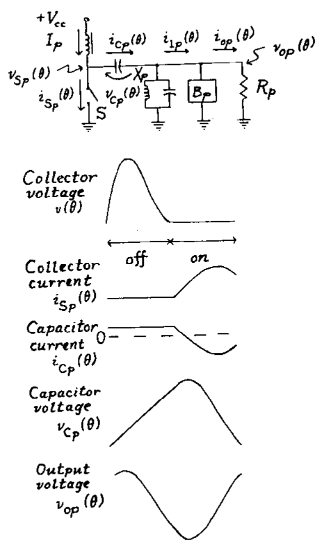

Fig. 12. Parallel-tuned class E amplifier with series capacitor.

circuit superficially resembles that of a class B amplifier, the transistor is operated as a switch, and the series capacitor absorbs the differences between the ideal switch and the ideal tuned output circuit. Note that this also differs from a saturated class B amplifier, in which the saturation resistance of the transistor absorbs the differences between the switch and the output circuit. Not all waveforms in this amplifier are the same as those in a series-tuned class E amplifier, which makes it more difficult to derive the analogy.

To evaluate this amplifier, let the output current be

$$i_{0p}(\theta) = \alpha \cos(\theta + \beta). \tag{5.2}$$

As before,

$$i_{1p}(\theta) = \alpha \cos(\theta + \beta) + \alpha R_p B_p \sin(\theta + \beta) \tag{5.3}$$

$$= \alpha_1 \cos(\theta + \beta_1). \tag{5.4}$$

TABLE III
ANALOGOUS PARAMETERS FOR PARALLEL-TUNED SERIES-CAPACITOR AMPLIFIER

Parallel-tuned amplifier		Series-tuned equivalent	
optimum	general	general	optimum
$\pi/2$	$y_p = \pi - y$	$y = \pi - y_p$	$\pi/2$
5.446	$X_p = \frac{1}{B}$	$B = \frac{1}{X_p}$	0.1836
29.66	$R_p = \frac{X_p^2}{R}$	$R = \frac{1}{B^2 R_p}$	1.0
0.0342	$B_p = \frac{X}{RR_p}$	$X = B_p R R_p$	1.152
49.052°	$\psi_p = \psi$	$\psi = \psi_p$	49.052°
−32.482°	$\beta = \varphi$	$\varphi = \beta$	−32.482°
0.576	$P_{op} = P_o$	$P_o = P_{op}$	0.576
0.576	$P_{ip} = P_i$	$P_i = P_{ip}$	0.576
1.000	$\eta_p = \eta$	$\eta = \eta_p$	1.000
0.1972	$\alpha = cB$	$c = \alpha X_p$	1.734
0.576	$I_p = I$	$I = I_p$	0.576

The collector voltage and current waveforms are derived in a manner analagous to that used in the series-tuned amplifier, and surprisingly have the same forms as those in the series-tuned amplifier. Since this is a parallel-tuned amplifier, the fundamental frequency component of the collector current must be equal to the fundamental frequency component of the fictitious output current $i_1(\theta)$, producing

$$\alpha_1 = \frac{1}{\pi}\int_0^{2\pi} i_{Sp}(\theta) \cos(\theta + \beta_1)\, d\theta \tag{5.5}$$

and

$$0 = \frac{1}{\pi}\int_0^{2\pi} i_{Sp}(\theta) \sin(\theta + \beta_1)\, d\theta. \tag{5.6}$$

Note that these equations are analogous to those which produced the functions g and h, with two exceptions. First, the function $i_{Sp}(\theta)$ is a current waveform, consisting of a dc value plus a sinusoid. Secondly, the sine and cosine are reversed. It is possible, however, to make analogies by observing that the current waveform is similar to a derivative of the voltage waveform. This should be no surprise, since the collector voltage in the series-tuned amplifier is produced by integrating a current waveform, and the current waveform in the parallel-tuned amplifier is produced by differentiating a voltage waveform.

The effect of differentiation on the fundamental frequency component is a 90° phase shift. This means that for the parallel-tuned amplifier, solving

$$c_1 = \frac{1}{\pi}\int_0^{2\pi} \frac{dv(\theta)}{d\theta} \cos(\theta + \varphi_1)\, d\theta \tag{5.7}$$

and

$$0 = -\frac{1}{\pi}\int_0^{2\pi} \frac{dv(\theta)}{d\theta} \sin(\theta + \varphi_1)\, d\theta \tag{5.8}$$

is equivalent to solving (2.16) and (2.20) for the series-tuned amplifier.

The method of making the analogy is now apparent, although many tedious derivations are omitted. Table III gives the resulting analogies between the parameters of this amplifier and those used in the series-tuned amplifier. Note that all of the waveforms have the familar shapes, with the exception of those of the series capacitor.

VI. Comments and Conclusions

The equations governing the operation of an idealized class E RF power amplifier have been derived. These equations were first used to determine the circuit elements required to obtain an efficiency of 100 percent, and then used to determine the optimum set of parameters for 100-percent efficient operation. The harmonic structure of the collector waveform was then determined, and analogies for related circuit configurations were presented. A brief comparison [3] of the optimum class E amplifier to other amplifiers is given in Table I.

The basic equations derived herein can also be used to determine the effects of such things as mistuning and frequency variation. The results of this solution [5] show the amplifier to be quite tolerant of variations in the circuit elements. Modification of the basic equations to include a shunt inductor could be used to produce a better understanding of class F (third harmonic resonator) power amplifiers. Class E frequency multipliers have been analyzed by similar techniques by Kozyrev [8]. The high-Q assumption used here might be avoided by using a more exact transient analysis method described by Liou [9].

Many modern single-ended-tuned solid-state VHF and UHF power amplifiers contain many aspects of class E operation, as shown in the models used by Abbott [10]. While these amplifiers are commonly called "class C," the presence of a series-tuned output circuit and a non-sinusoidal collector voltage waveform [11] shows that they do not resemble classical class C operation. Thus, while the theory given in this paper is most applicable to the design of highly efficient RF power amplifiers, it may provide some badly needed insight into the operation of present VHF and UHF amplifiers. Accurate description of VHF and UHF amplifiers will require modification of the basic equations presented here to include varactor capacitance and additional circuit elements to account for lead and stray reactances.

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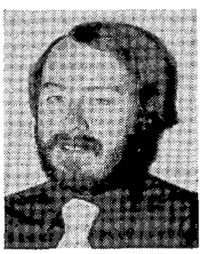

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